Arc 1

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Given a circular arc q, find the radius r and the angle θ subtended by q at the center M of circle k.

Solution:

Let q be the arc of a circle k with unknown center M and unknown radius r. Let p_0 and p_1 be the end points of q. Let p_2 be the point halfway between p_0 and p_1. Let p_3 be the intersection of q and the perpendicular to the line p_1p_0 through p_2. Let p_4 be the point halfway between p_0 and p_3. Let p_5 be the intersection of q and the perpendicular to p_0p_3 through p_4. The lines p_3p_2 and p_5p_4 intersect at M. Let a be the distance from p_3 to p_2, i.e., the magnitude of $p_3 - p_2$, b the distance from p_0 to p_2, and c the distance from p_0 to p_3. Let α be $\angle p_3p_0p_2$ and β be $\angle p_0p_3p_2$. Then $\alpha = \sin(a/c)$ and $\beta = \sin(b/c)$. Let γ be $\angle p_4Mp_3$. Since $\angle p_3p_4M$ is a right angle, $\gamma = \alpha$. Since the arc p_0p_5 is a quarter of the arc p_0p_1, the angle θ subtended by q at M is 4γ.